Radiatively Induced Neutrino Masses and Large Higgs-Neutrino Couplings in the Standard Model with Majorana Fields

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ABSTRACT

The Higgs sector of the Standard Model (SM) with one right-handed neutrino per family is systematically analyzed. In a model with intergenerational independent mixings between families, we can account for very light neutrinos acquiring Majorana masses radiatively at the first electroweak loop level. We also find that in such a scenario the Higgs coupling to the light–heavy neutrinos and to the heavy–heavy ones may be remarkably enhanced with significant implications for the production of these heavy neutrinos at high energy colliders.

1 Introduction

One of the outstanding problems in particle and astrophysics is connected with the question of the neutrino mass, which if nonzero, has to be very small in the range of 10 eV for cosmological reasons [1]. Such small neutrino masses could also eventually account for most of the dark matter in the universe [2,3]. Also, small mass differences between neutrinos ($\Delta m_\nu \sim 10^{-2} - 10^{-6}$) may resolve the Solar neutrino problem [4]. Finally, double-beta decay experiments seem also to favour neutrino masses $\stackrel{<}{\sim} 10$ eV [5]. In the minimal SM, these particles are taken to be massless. In other extensions of it, a desired nonzero Majorana mass can be obtained by including right-handed neutrino fields, where the well known "see-saw" mechanism takes place generating very light neutrinos [6]. In most theories, the mass of the light neutrinos ν_i is related to that of the heavy ones N_i via $m_\nu \sim m_D^2/m_N$, where m_D is the Dirac mass scale. Therefore, in order to naturally provide very small neutrino masses, one has to impose a very large scale on m_N (e.g. $10^7 - 10^8$ GeV, for $m_D \sim m_{leptons}$ and $m_\nu \leq 10 - 40$ eV) [7]. However, we will show that large intergenerational indepedent mixings of the order of 0.1 may give rise to $m_\nu \leq 10$ eV at the first electroweak loop level, with m_N being in the 100 GeV range. These heavy neutrinos can copiously be produced in the forthcoming e^+e^- or hadron colliders with typical signals being like-sign dilepton pairs and additional jets with no missing transverse momentum p_T [8]. We also find that in some scenarios the Higgs coupling to νN and NN can significantly contribute to the production of these heavy neutrinos.

This work has been organized as follows: In section 2 we give the description of the SM in which one right-handed neutrino field for each family has been introduced. We carefully study the Higgs sector of the model and will give some constraints on the mixing parameters involved. In section 3 we elaborate an illustrative example considering the case where only two neutrino species mix with each other. In particular, we present the main theoretical features of the mass matrices that describe two massless neutrinos at the tree level. In sections 4 and 5 we calculate the radiatively induced Majorana masses for the light neutrinos and address numerically the issue of the implications of large $H - \nu - N$ and $H - N - N$ couplings for the production cross sections of the heavy neutrinos, respectively. Finally, in section 6 we summarize our conclusions.

2 The Standard Model with right-handed neutrinos

Let us start the discussion by giving a general description of the Yukawa sector of the

SM with right-handed neutrinos. After spontaneous symmetry breaking the relevant part of the Lagrangian containing Dirac and Majorana mass terms is given by

$$- L_{mass}^{\nu} = \overline{\nu}_{R_i}^0 m_{D_{ij}}^{\dagger} \nu_{L_j}^0 + \overline{\nu}_{L_i}^0 m_{D_{ij}} \nu_{R_j}^0 + \frac{1}{2}\overline{\nu}_{R_i}^{0C} m_{M_{ij}}^{\dagger} \nu_{R_j}^0 + \frac{1}{2}\overline{\nu}_{R_i}^0 m_{M_{ij}} \nu_{R_j}^{0C} \qquad (2.1)$$

In eq. (1) m_D and m_M are Dirac and Majorana $n_G \times n_G$ mass matrices, respectively and $\nu_L^0 (\nu_R^0)$ is the left(right)-handed Weyl spinor which describes the neutrino field. Note that L_{mass}^{ν} has the most general form which is invariant under the gauge transformation $SU(2)_L \otimes U(1)_Y$. In other words, in (2.1) we assume the absence of isotriplet Higgs scalars and Majoron fields [9]. Thus, we can now express L_{mass}^{ν} in terms of the Majorana fields

$$\begin{aligned} f &= \nu_L^0 + (\nu_L^0)^C \\ F &= \nu_R^0 + (\nu_R^0)^C \end{aligned} \qquad (2.2)$$

as follows:

$$- L_{mass}^{\nu} = \frac{1}{2}(\overline{f}_L, \overline{F}_L)_i M_{ij}^{\nu} \begin{pmatrix} f_R \\ F_R \end{pmatrix}_j \quad + \quad h.c. \qquad (2.3)$$

with

$$M^{\nu} = \begin{pmatrix} 0 & m_D \\ m_D^{\mathrm{T}} & m_M \end{pmatrix} \qquad (2.4)$$

Applying the properties of Majorana fields to eq. (2.3), we find that M^{ν} is generally a complex symmetric matrix ($M^{\nu} = M^{\nu \mathrm{T}}$). The matrix M^{ν} can be diagonalized by a $2n_G \times 2n_G$ unitary matrix U^{ν} in the following way:

$$U^{\nu T} M^{\nu} U^{\nu} = \hat{M}^{\nu} \qquad (2.5)$$

At the same time, the Majorana fields have to be transformed according to

$$\begin{pmatrix} f_R \\ F_R \end{pmatrix} = U^{\nu} \begin{pmatrix} \nu_R \\ N_R \end{pmatrix} \quad , \quad \begin{pmatrix} f_L \\ F_L \end{pmatrix} = U^{\nu *} \begin{pmatrix} \nu_L \\ N_L \end{pmatrix} \qquad (2.6)$$

Diagonalizing the mass matrix M^{ν}, we obtain n_G light neutrinos (ν_i) and n_G heavy ones (N_i). This and other constraints on the structure of m_D, m_M and neutrino mixings, which we shall give below, are imposed by phenomenology.

We are now in the position to give explicitly the Lagrangian which describes the interaction between Majorana neutrinos and gauge or Higgs bosons in terms of mass eigenstates.

$$L_{int}^{W - \nu_M - l} = -\frac{g_W}{2\sqrt{2}} W^{-\mu} \Big[\overline{l}_i \gamma_{\mu} \gamma_{-} B_{l_i \nu_j} \nu_j + \overline{l}_i \gamma_{\mu} \gamma_{-} B_{l_i N_j} N_j \Big] \quad + \quad h.c.$$

$$\gamma_- = 1 - \gamma_5 \tag{2.7}$$

$$
\begin{aligned}
L_{int}^{Z-\nu_M-\nu_M} = &-\frac{g_W}{4\cos\theta_W}Z^{0\mu}\Big[\ \overline{\nu}_i\gamma_\mu[iIm(C_{\nu_i\nu_j}) - \gamma_5 Re(C_{\nu_i\nu_j})]\nu_j \\
&+ (\overline{\nu}_i\gamma_\mu[iIm(C_{\nu_iN_j}) - \gamma_5 Re(C_{\nu_iN_j})]N_j \quad + \quad h.c.) \\
&+ \overline{N}_i\gamma_\mu[iIm(C_{N_iN_j}) - \gamma_5 Re(C_{N_iN_j})]N_j\ \Big]
\end{aligned}
\tag{2.8}
$$

$$
\begin{aligned}
L_{int}^{H-\nu_M-\nu_M} = &-\frac{g_W}{4M_W}H^0\Big[\ \overline{\nu}_i[(m_{\nu_i} + m_{\nu_j})Re(C_{\nu_i\nu_j}) + i\gamma_5(m_{\nu_j} - m_{\nu_i})Im(C_{\nu_i\nu_j})]\nu_j \\
&+ 2\overline{\nu}_i[(m_{\nu_i} + m_{N_j})Re(C_{\nu_iN_j}) + i\gamma_5(m_{N_j} - m_{\nu_i})Im(C_{\nu_iN_j})]N_j \\
&+ \overline{N}_i[(m_{N_i} + m_{N_j})Re(C_{N_iN_j}) + i\gamma_5(m_{N_j} - m_{N_i})Im(C_{N_iN_j})]N_j\ \Big]
\end{aligned}
\tag{2.9}
$$

The matrices B and C given above can be expressed in terms of U^ν by

$$B_{l_ij} = \sum_{k=1}^{n_G} V_{ik}^l U_{kj}^{\nu*} \qquad \text{with} \quad j = 1,\dots,2n_G \tag{2.10}$$

$$C_{ij} = \sum_{k=1}^{n_G} U_{ki}^\nu U_{kj}^{\nu*} \qquad \text{with} \quad i,j = 1,2,\dots,2n_G \tag{2.11}$$

where V^l in eq. (2.10) is the corresponding Kobayashi-Maskawa (KM) matrix for the lepton sector. For a proper labeling of the neutrino fields in eqs (2.10), (2.11), notice that the indices i, j refer for ν_i or ν_j to $i, j = 1,\dots,n_G$ and for N_i or N_j to $i, j = n_G + 1,\dots,2n_G$. Furthermore, the matrices B, C obey the following equalities:

$$\sum_{k=1}^{2n_G} B_{l_ik}B_{l_jk}^* = \delta_{l_il_j} \tag{2.12}$$

$$\sum_{k=1}^{n_G} B_{l_kj}B_{l_ki}^* = C_{ij} \quad \text{with} \quad i,j = 1,\dots,2n_G \tag{2.13}$$

$$\sum_{k=1}^{2n_G} C_{ik}C_{jk}^* = C_{ij} \quad \text{with} \quad i,j = 1,\dots,2n_G \tag{2.14}$$

Eq. (2.12) can be regarded as the generalized form of the unitarity condition for n_G charged leptons and $2n_G$ neutrinos. Since in section 4 we will calculate neutrino masses induced by loop corrections in the Feynman gauge, we, for definiteness, give the relevant couplings of the Majorana neutrinos with the unphysical Goldstone bosons χ^- and χ^0.

$$L_{int}^{\chi^--\nu_M-l} = -\frac{g_W}{2\sqrt{2}M_W}\chi^-\Big[\ \overline{l}_i[m_{l_i}B_{l_i\nu_j}\gamma_- - \gamma_+ B_{l_i\nu_j}m_{\nu_j}]\nu_j$$

$$+ \bar{l}_i[m_{l_i}B_{l_iN_j}\gamma_- - \gamma_+ B_{l_iN_j}m_{N_j}]N_j \Big] \quad + \quad h.c. \tag{2.15}$$

$$
\begin{aligned}
L_{int}^{\chi^0-\nu_M-\nu_M} &= -\frac{ig_W}{4M_W}\chi^0\Big[\, \bar{\nu}_i[(m_{\nu_i}+m_{\nu_j})\gamma_5 Re(C_{\nu_i\nu_j}) + i(m_{\nu_j}-m_{\nu_i})Im(C_{\nu_i\nu_j})]\nu_j \\
&+ 2\bar{\nu}_i[(m_{\nu_i}+m_{N_j})\gamma_5 Re(C_{\nu_iN_j}) + i(m_{N_j}-m_{\nu_i})Im(C_{\nu_iN_j})]N_j \\
&+ \overline{N}_i[(m_{N_i}+m_{N_j})\gamma_5 Re(C_{N_iN_j}) + i(m_{N_j}-m_{N_i})Im(C_{N_iN_j})]N_j \,\Big]
\end{aligned}
\tag{2.16}
$$

Up to now we have treated the neutrinos as nonzero mass particles and therefore our Lagrangians derived above are of general use. One can now easily prove the following theorem:

A sufficient and necessary condition for the j-th light neutrino to be massless already at tree level is

$$\sum_{k=1}^{2n_G} M_{ik}^\nu U_{kj}^\nu = 0 \quad \text{for each} \quad i = 1, 2, \ldots, 2n_G \tag{2.17}$$

Assuming now all light neutrinos ν_i to be massless, one finds that couplings proportional to m_{ν_i} disappear in the Lagrangians (2.9), (2.15) and (2.16). Nevertheless, what kind of structure of M^ν follows from restriction (2.17), will be seen in the context of a two generation model ($n_G = 2$) in section 3.

Sometimes, in order to avoid excessive complication in our calculations, we expand U^ν in power series of the matrix parameter $\xi = m_D m_M^{-1}$ [10], with the constraint $\xi_{ij} < 1$. The form of U^ν to the third order of ξ can be estimated to be

$$U^\nu = \begin{pmatrix} 1 - \frac{1}{2}\xi^*\xi^T & \xi^*(1-\frac{1}{2}\xi^T\xi^*)J \\ -\xi^T(1-\frac{1}{2}\xi^*\xi^T) & (1-\frac{1}{2}\xi^T\xi^*)J \end{pmatrix} \quad + \quad \mathcal{O}(\xi^4) \tag{2.18}$$

where J is a $n_G \times n_G$ diagonal unitary matrix, which guarantees that the nonzero mass eigenvalues have positive values. Up to next to leading order in ξ the neutrino masses are given by

$$m_\nu = m_D\xi^T = \xi m_D^T = 0 \tag{2.19}$$

$$m_N = Jm_M[\, 1 + \frac{1}{2m_M}(\xi^\dagger m_D + m_D^T\xi^*) + \mathcal{O}(\xi^3)\,]J \tag{2.20}$$

Making also use of eq. (2.18) we find that the matrices B, C in this approximation are determined by

$$B_{l_i\nu_j} = [V^l(1 - \frac{1}{2}\xi\xi^\dagger)]_{l_i\nu_j} \quad , \quad B_{l_iN_j} = [V^l\xi(1-\frac{1}{2}\xi^\dagger\xi)J^*]_{l_iN_j} \tag{2.21}$$

$$C_{\nu_i\nu_j} = (1-\xi\xi^\dagger)_{\nu_i\nu_j} \ , \quad C_{\nu_iN_j} = [\xi(1-\xi^\dagger\xi)J^*]_{\nu_iN_j} \ , \quad C_{N_iN_j} = [J\xi^\dagger\xi J^*]_{N_iN_j} \tag{2.22}$$

The mixing couplings $C_{\nu_i \nu_j}$ can be taken to be diagonal, because the masslessness condition (2.17) gives the freedom to rotate the light neutrino fields by an arbitrary unitary matrix R as

$$\nu'_i = R_{ij}\nu_j \tag{2.23}$$

The matrix C spanned in the massless neutrino space is hermitian and can hence be diagonalized by choosing the unitary matrix R appropriately.

The allowed values of the mixing parameters ξ_{ij} have been systematically investigated in [11], where a global analysis based on charge-current universality, neutral-current effects and other experimental constraints has been performed. In fact, it has been shown that these parameters have maximal values of the order of $0.1 - 0.2$, where the larger bound applies safely to the systems $e - \tau$, $\mu - \tau$ [12]. Such large mixings are even preferred by some neutrino-mass schemes [13] for resolving the solar-neutrino problem through the MSW mechanism [4]. On the other hand, heavy neutrinos N_i with mixings $\xi_{ij} > 3 \ 10^{-2}$ must be heavier than the Z^0 boson, since otherwise they could already be produced in Z^0 decays at LEP.

For the $H^0 - \nu - N$ and $H^0 - N - N$ couplings some comments are in order here. These interactions are significantly enhanced for heavy neutrinos with $m_N \gg M_W$. From eqs (2.8) and (2.9) we remark that the $H^0 - \nu - N$ coupling is almost by a factor m_N/M_W larger than the $Z^0 - \nu - N$ coupling, while the $H^0 - N - N$ coupling is $2m_N/M_W$ times as large as the Z^0 corresponding one (i.e. $Z^0 - N - N$). This fact may have important implications for the production cross sections of these heavy neutral leptons at high energies [14]. We will address this issue numerically in section 5. Finally, it should be also noted that the Dirac mass terms $m_{D_{ij}}$ defined in (2.1) cannot be arbitrarily large, since they are constrained by renormalization-group-triviality bounds ($m_{D_{ij}} \lesssim 0.3$ TeV) [15].

3 The neutrino mass matrix − the case $n_G = 2$

For the sake of illustration, we will now give the main theoretical characteristics for the mass matrices describing two massless neutrinos only ($n_G = 2$). The general form of the matrices m_D and m_M as parameterized in [16] is given by

$$m_D = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad m_M = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} \tag{3.1}$$

where m_D is a general complex matrix and m_M can be chosen to be real. Now, the requirement for two zero eigenvalues corresponding to the two massless neutrinos prescribes

that M^ν fulfills the following two conditions:

$$\prod_{i=1}^{4} m_i^2 \;=\; \det(M^\nu M^{\nu\dagger}) = 0 \tag{3.2}$$

$$\sum_{i<j<k}^{4} m_i^2 m_j^2 m_k^2 \;=\; \frac{1}{6}\Big[tr^3(M^\nu M^{\nu\dagger}) + 2tr(M^\nu M^{\nu\dagger})^3 - 3tr(M^\nu M^{\nu\dagger})tr(M^\nu M^{\nu\dagger})^2\Big] = 0$$

$$\tag{3.3}$$

Equation (3.2) leads automatically to the constraint:

$$\det m_D \;=\; 0 \tag{3.4}$$

In particular, assuming without any loss of generality that $a \neq 0$, we can derive from eqs (3.3) and (3.4) that

$$d = \frac{bc}{a} \quad , \qquad B = -\frac{b^2}{a^2}A \tag{3.5}$$

We also find that the parameters a, b ,c ,d should be either purely real or purely imaginary numbers. Taking, for example, these parameters to be real, we can evaluate the masses of the heavy neutrinos to be

$$m_{N_1} \;=\; \frac{A}{2}\Big[1 - \frac{b^2}{a^2} + \Big(1 + \frac{b^2}{a^2}\Big)\sqrt{1 + \frac{4a^2}{a^2 + b^2}\Big(1 + \frac{c^2}{a^2}\Big)\frac{a^2}{A^2}}\,\Big] = A + \mathcal{O}(1/A)$$

$$m_{N_2} \;=\; -\frac{A}{2}\Big[1 - \frac{b^2}{a^2} - \Big(1 + \frac{b^2}{a^2}\Big)\sqrt{1 + \frac{4a^2}{a^2 + b^2}\Big(1 + \frac{c^2}{a^2}\Big)\frac{a^2}{A^2}}\,\Big] = \frac{b^2}{a^2}A + \mathcal{O}(1/A)$$

$$\tag{3.6}$$

In this scheme the matrix J turns out to be

$$J = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \tag{3.7}$$

Notice that even in case $a = b = c$, we have $d = a$ and $B = -A$, which means that models with family-independent mixings can naturally account for massless neutrinos already at the tree level. In reality one has to assume that the Dirac mass terms are described by a universal Yukawa coupling a and the two right-handed weak eigenstates $\nu^0_{R_{1,2}}$ possess opposite CP quantum numbers. However, in our forthcoming calculations we will consider that there exist small perturbations on this family-independent scenario in such a way that eq. (3.5) is always valid. Recently, patterns with "democratic"-type mixing between quark families have also been proposed in [17] in order to explain the structure of the usual KM-mixing matrix.

So far, the "see-saw" mechanism is the only known scheme [6,7] for naturally generating small, nonvanishing, neutrino masses. According to this mechanism, the Majorana mass terms ($m_M \sim A \sim m_N$) in eq. (3.1) can be regarded as remnant parts of a more fundamental theory, which contains the SM as an effective low energy approximation. Some examples could be the Left–Right symmetric models [18] or grand unified models (GUT), e.g. $SO(10)$ [19], or more complicated patterns arising from certain embeddings into the gauge group E_6 [19]. In all these models it is possible to have TeV-mass scales determined by the breaking mechanism itself, as the symmetry of the original gauge group breaks spontaneously down to $SU(2)_L \otimes U(1)_Y$. In some GUT theories one also obtains that the Dirac mass matrix of the neutrinos equals, up to a factor of proportionality of the order one, the quark or the charged-lepton mass matrix [6,19]. Thus, in order to have $m_{\nu_i} \lesssim 1 - 10$ eV for $m_D \approx 1$ GeV through the "see-saw" mechanism, one must require that very heavy neutrinos with masses $m_{N_i} \gtrsim 10^7 - 10^8$ GeV are present. However, as we will see in the next section, in the context of the model outlined above small neutrino masses m_ν can naturally be induced by radiative corrections with $m_{N_i} \sim 100$ GeV and $\xi_{ij} \sim 0.1$.

4   Radiatively induced neutrino masses

We will now proceed with the calculation of the Feynman graphs depicted in fig. 1, which give rise to one-loop neutrino masses of the Majorana type. Note that only the heavy–light neutrino couplings with the Higgs and Z^0 boson are responsible for the generation of a nonzero neutrino mass matrix given by

$$m^\nu_{\nu_i \nu_j} \;=\; \Sigma^{\nu_i \nu_j}(\slashed{q})\Big|_{\slashed{q}=0} \tag{4.1}$$

where $\Sigma^{\nu_i \nu_j}(\slashed{q})$ is the selfenergy diagram of neutrinos. In particular, working in the on-shell renormalization scheme and adopting the Feynman gauge [20], we find that the graphs (1d)-(1f) do not contribute to m^ν. The graph (1b) is finite by itself, while the ultraviolet divergences existing in (1a) and (1c) cancel each other. Thus, we finally arrive at the following expression for the neutrino mass matrix:

$$m^\nu_{\nu_i \nu_j} \;=\; \frac{\alpha_W}{16\pi} \sum_{k=1}^{n_G} \frac{m_{N_k}}{M_W^2} C_{\nu_i N_k} C_{\nu_j N_k} F(m_{N_k}^2, M_Z^2, M_H^2) \tag{4.2}$$

with

$$F(m_{N_k}^2, M_Z^2, M_H^2) \;=\; m_{N_k}^2 [f(m_{N_k}^2, M_Z^2) - f(m_{N_k}^2, M_H^2)] - 4M_Z^2 f(m_{N_k}^2, M_Z^2) \tag{4.3}$$

where the function $f(m_N^2, M^2)$ is defined as

$$f(m_N^2, M^2) = \frac{m_N^2}{m_N^2 - M^2} \ln \frac{m_N^2}{M^2} + \ln \frac{M^2}{\mu^2} - 1 \tag{4.4}$$

Although f is a μ-dependent function, one can, however, show that the final result in eq. (4.2) does not explicitly depend on the subtraction point μ. To see that, we list the following useful identities:

$$\sum_{k=1}^{n_G} m_{N_k} C_{iN_k} C_{jN_k} = 0 , \quad \text{for} \quad i, j = 1, 2, \ldots, 2n_G \tag{4.5}$$

$$\sum_{k=1}^{n_G} m_{N_k} B_{l_i N_k} B_{l_j N_k} = 0 \tag{4.6}$$

Employing eq. (4.5), we immediately find that the μ-dependence existing in the last term of eq. (4.3) drops out. Eqs (4.5) and (4.6) also tell us that in the limit where all heavy neutrinos are degenerated, m^ν approaches zero. Consequently, the smallness of the n_G light neutrino masses can be attributed to the fact that there exist n_G nearly degenerated heavy neutrinos. Another important conclusion one can draw here is that eq. (4.5) does not impose any restriction on the masses of N_i. Their values can be obtained in connection with the phenomenologically constrained mixing parameters $(\xi \xi^\dagger)_{\nu_i \nu_j}$ [11]. To be specific, let us consider a model with two generations only ($n_G = 2$). Then, in the leading order of ξ, m^ν takes the simple form:

$$m^\nu = \frac{\alpha_W}{4\pi} \frac{M_H^2 + 3M_Z^2}{M_W^2} \ln \frac{|a|}{|b|} \frac{1}{A} \begin{pmatrix} a^2 & ac \\ ac & c^2 \end{pmatrix} \tag{4.7}$$

In order to obtain the mass eigenvalues, we use the freedom of rotating the neutrino fields at the tree level (see eq. (2.23)). Since $\det m^\nu = 0$, this implies that one neutrino is massless, while the mass of the other one can be obtained by

$$m_{\nu_2} = [tr(m^\nu m^{\nu\dagger})]^{\frac{1}{2}} \tag{4.8}$$

At this loop level, the above situation is also true when one considers the full expression of m^ν. Since in this two generation model m^ν is proportional to the form:

$$m^\nu_{\nu_i \nu_j} \propto C_{\nu_i N_2} C_{\nu_j N_2} \tag{4.9}$$

the $\det m^\nu$, with m^ν representing a tensor product of two vectors, will always vanish. However, the above mass hierarchy is no longer valid, when one introduces a third generation neutrino in the discussion. In that case, the radiative neutrino mass can be written down as follows:

$$m^\nu = [g(m_{N_2}) - g(m_{N_1})]m_{N_2} C_{\nu_i N_2} C_{\nu_j N_2} + [g(m_{N_3}) - g(m_{N_1})]m_{N_3} C_{\nu_i N_3} C_{\nu_j N_3} \tag{4.10}$$

where $g(m_{N_i})$ are some functions that can be computed from eq. (4.2). So, if in eq. (4.10), for example, we set $m_{N_1} = m_{N_3}$, then we recover the mass spectrum of the model with $n_G = 2$ discussed previously. One can therefore conclude that the neutrino mass hierarchy will be controlled by the heavy neutrino masses m_{N_i}. An extensive analysis for the more complicated case of three generation models will be given elsewhere [14,26].

To get an idea of some possible numerical values of m^ν in eq. (4.7), let

$$a = c = 10 \text{ GeV} \quad \text{and} \quad A = 100 \text{ GeV} \tag{4.11}$$

Then, for sufficiently small values of the perturbation parameter $\varepsilon = (b-a)/a$, e.g. $\varepsilon \sim 10^{-4}$, we get $m_{\nu_2} \sim 1 - 10$ eV ($M_H = 100$ GeV). This neutrino mass is also consistent with the cosmological requirement [1] that

$$\sum_{i=1}^{n_G} m_{\nu_i} \overset{<}{\sim} 40 \text{ eV} \tag{4.12}$$

Also, from eq. (3.6) we easily derive $m_N \simeq 102$ GeV.

To further illuminate this scenario, let us numerically investigate the mixing couplings $(\xi\xi^\dagger)_{\nu_i\nu_j}$. They have the form:

$$\xi\xi^\dagger = \frac{2|a|^2}{A^2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{4.13}$$

Using again the freedom of redefining the neutrino fields, $\xi\xi^\dagger$ can be diagonalized with eigenvalues given by

$$(\xi\xi^\dagger)_{diag.} = \frac{4|a|^2}{A^2}(0, \ 1) \tag{4.14}$$

According to [12], the lowest upper bound on $a/A \overset{<}{\sim} 0.2$ can be obtained from the τ-neutrino mixings, whereas similar bounds coming from ν_μ, ν_e are more stringent ($a/A \overset{<}{\sim} 0.1$).

5 Phenomenological implications of large $H^0 - \nu - N$ and $H^0 - N - N$ couplings

In this section we will discuss the phenomenological implications of our model for the production of heavy neutral leptons and the associated phenomena of lepton-number violation. To gauge the possibility of measuring such effects at the next generation colliders (LHC, SSC, INP, etc.), we obtain numerical results for production cross sections of heavy

neutrinos N_i – paying more attention on the Higgs-mediated processes. Since there exist a variety of works studying similar phenomena [8,10], one may hence consider in part the discussion given here as complementary to them.

In order to be able to discuss the production of heavy neutrinos via a Higgs boson, we summarize all H^0-production cross sections at hadron and e^+e^- machines in fig. 2. For numerical estimates we have used $EHLQ$ parton distribution functions (set 2) [22]. The subsequent decay rate of the heavy neutrinos into a specific final state can be described by their partial widths and branching ratios and, as given in [10], they are

$$\Gamma(N \to l^\pm W^\mp) = \frac{\alpha_W}{16M_W^2}|B_{lN}|^2 m_N^3(1 + \frac{2M_W^2}{m_N^2})(1 - \frac{M_W^2}{m_N^2})^2\theta(m_N - M_W) \quad (5.1)$$

$$\Gamma(N \to \nu Z^0) = \frac{\alpha_W}{16M_W^2}|C_{\nu N}|^2 m_N^3(1 + \frac{2M_Z^2}{m_N^2})(1 - \frac{M_Z^2}{m_N^2})^2\theta(m_N - M_Z) \quad (5.2)$$

However, in case $m_N > M_H$ another decay channel will be opened kinematically, given by the partial width

$$\Gamma(N \to \nu H^0) = \frac{\alpha_W}{16M_W^2}|C_{\nu N}|^2 m_N^3(1 - \frac{M_H^2}{m_N^2})^2\theta(m_N - M_H) \quad (5.3)$$

The branching ratio for a situation where $m_N \gg M_W, M_Z, M_H$ and $C_{\nu N} \simeq B_{lN}$ is

$$\sum_{i=1}^{n_G} Br(N \to \nu_i Z^0) = \sum_{i=1}^{n_G} Br(N \to l_i^+ W^-) = \sum_{i=1}^{n_G} Br(N \to \nu_i H^0) = \frac{1}{4} \quad (5.4)$$

Instead, if $m_N \stackrel{<}{\sim} M_H$ the above ratio for the different decay modes increases up to 1/3.

The Higgs-mediated processes at high energies can produce heavy neutrinos N_i through the $H^0 - N - N$ and $H^0 - \nu - N$ couplings. The Majorana nature of the heavy neutrinos in the first class of reactions (i.e. e^+e^-, $pp \to H^{0*} \to NN$) may be proved by detecting like-sign dilepton pairs associated with jets with no missing p_T [23]. The second class of processes (i.e. e^+e^-, $pp \to H^{0*} \to N\nu$) will be more problematic. Nevertheless, if the standard background contributed to Higgs decays could be theoretically removed, the detection of neutral leptons via a flavour-nonconserved $H^0 - N - \nu$ coupling would be favourably interpreted as an indication of heavy Majorana neutrino events. Another feature of these processes is the very large missing p_T at the Higgs-resonance line. Since we are interested in the production rate of N_i via a heavy Higgs boson, let us, for completeness, quote the main partial decay widths.

$$\Gamma(H^0 \to \nu_i N_j) = \frac{\alpha_W M_H}{8}|C_{\nu_i N_j}|^2\frac{m_{N_j}^2}{M_W^2}(1 - \frac{m_{N_j}^2}{M_H^2})\theta(M_H - m_{N_j}) \quad (5.5)$$

$$\Gamma(H^0 \to N_i N_j) = \delta_{ij}\frac{\alpha_W M_H}{4}(ReC_{N_i N_j})^2\frac{m_{N_i}^2}{M_W^2}\left[1 - 4\frac{m_{N_i}^2}{M_H^2}\right]^{\frac{3}{2}}\theta(M_H - m_{N_i} - m_{N_j}) \quad (5.6)$$

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In eq. (5.6) we have assumed that N_i are nearly degenerated as dictated by this model. In addition, the phenomenology of the Higgs boson in the SM is studied rather extensively at present [24,25]. Fig. 3 shows that for $m_N \simeq 100$ GeV the channel $H^0 \rightarrow \nu N$ could be the most dominant decay mode in the intermediate Higgs mass range (i.e $100 \leq M_H \leq 150$ GeV). Such events are characterized by a very large missing transverse momentum, a fact that can probably be exploited to reduce efficiently the contributing background *. In particular, when B_{lN} is suppressed but $C_{\nu N}$ not [10], the reaction $e^+ e^- \rightarrow H^{0*} \rightarrow \nu N X$ will become the most significant production mechanism for 100 GeV neutrinos with a cross section value $\sigma_{tot} \simeq 0.3$ pb at $c.m.s.$ energies $\sqrt{s_{tot}} = 1 - 2$ TeV.

The branching ratios for the decay of the Higgs scalar into the modes NN and νN neutrinos are generally given in figs 4 and 5. More precisely, for $M_H \simeq 3m_N$, we get a maximum value for $Br(H \rightarrow NN) \simeq 1.6 \ 10^{-3}$ and for $M_H \simeq 1.5m_N$ we have a maximum value for $Br(H \rightarrow N\nu) \simeq 3 \ 10^{-2}$. Illustratively, we mention that for $m_N = 150$ GeV, $M_H \simeq 450$ GeV and $a/A = 0.2$ we expect about 480 lepton-violating events per year at LHC ($\sqrt{s_{tot}} = 16$ TeV) assuming the standard high luminosity $\mathcal{L} = 4 \ 10^5 pb^{-1}$/year. The corresponding rate at SSC ($\sqrt{s_{tot}} = 40$ TeV, $\mathcal{L} = 10^4 \ pb^{-1}$/yr) is relative small, i.e. about 80 equal-sign dileptons a year.

To have a complete picture, in fig. 6 we give the cross-section values of the W-mediated tree process, relevant for the production of dilepton signals with no missing p_T at pp machines, as a function of m_N. The production cross section is evaluated by

$$\sigma(s_{tot}) = 2 \int dx \int dy [f_u^p(x, Q^2) f_d^p(y, Q^2) + f_c^p(x, Q^2) f_s^p(y, Q^2)] \hat{\sigma}(\hat{s}) \qquad (5.7)$$

where f^p's are parton distribution functions [22] at $Q^2 = \hat{s} = xy s_{tot}$ and x, y are usual kinematical variables restricted to the intervals

$$\frac{m_N^2}{s_{tot}} \leq x \leq 1 \quad , \quad \frac{m_N^2}{x s_{tot}} \leq y \leq 1 \qquad (5.8)$$

Moreover, the subprocess cross section $\hat{\sigma}$ is given by

$$\hat{\sigma}(\hat{s}) = \frac{\pi \alpha_W^2}{72 \hat{s}^2 (\hat{s} - M_W^2)^2} |B_{lN}|^2 (\hat{s} - m_N^2)^2 (2\hat{s} + m_N^2) \qquad (5.9)$$

Now, we can easily find that for unsuppressed $W - l - N$ couplings (e.g. $B_{lN} \simeq C_{\nu N}$), this scattering process gives a large amount of lepton-number-violating signals. Specifically, for $m_N = 150$ GeV one could expect up to 10000(1000) events with no missing p_T at

*Similar technics relying on particular kinematical cuts are used, for example, for the reconstruction of the production of supersymmetric particles at hadron colliders [27]. In any case, the viability of such heavy neutrino signals from the background will be investigated in [14].

$LHC(SSC)$. As a consequence, high energy colliders will successfully explore such heavy neutrino scenarios for the first time and may indirectly lead us to some important clues concerning the nature of ordinary neutrinos.

6 Conclusions

We have presented a new radiative mechanism of generating small neutrino masses, in the simplest model which predicts heavy Majorana neutrinos, i.e. the SM with one right-handed neutrino per family. This mechanism based on large intergeneration mixings of "democratic" type ($\xi_{\nu N} \sim 0.1$) naturally provides very light neutrinos with $m_\nu \stackrel{<}{\sim} 10$ eV at the first electroweak loop level and nearly degenerated heavy neutrinos N_i with m_N being in the 100 GeV range. On the contrary, from the ordinary "see-saw" mechanism and taking $m_{D_i} \sim m_{l_i}$, one derives the values $\sim 10^{-7}$ eV, 10^{-2} eV, 10 eV for m_{ν_e}, m_{ν_μ}, m_{ν_τ}, respectively, with a heavy neutrino mass $m_N \sim 10^9$ GeV. The common feature of such cosmologically consistent scenarios proposed by many authors [7] is that they generally require an extremely large Majorana scale ($m_N \sim m_M \sim 10^8 - 10^{12}$ GeV) and are mostly associated with invisible axions. However, our suggested scheme is a rather interesting and viable possibility which implies heavy neutral leptons with a rather low mass scale $m_M \sim 100$ GeV.

In section 5, we have discussed the phenomenological implications of the model under consideration for the production of heavy neutrinos N_i and the associated lepton-number violating signatures. Paying special attention on the Higgs sector of the model, we have found numerically that Higgs bosons may predominantly decay into heavy Majorana neutrinos with $m_N \simeq 100$ GeV for $100 \leq M_H \leq 150$ GeV. Furthermore, for heavier neutrino masses the Higgs-mediated processes at LHC can give rise to a sufficiently large number of like-sign dileptons with no missing p_T of the order of 10^3 events a year. For comparison, we have also numerically evaluated the tree-level W-exchange process, $pp \to W^{-*} \to l^- N X$ (see also fig. 6). This provides an event rate of lepton-number-violating signals up to one hundred times larger than the H^0-exchange processes, when an unsuppressed $W - l - N$ coupling is assumed.

A large $H - \nu - N$ coupling may also have important consequences on obtaining large lepton-flavour-nonconservation phenomena in H^0 decays [26]. Such signals characterized by no missing p_T and no hard jet events can easily be reconstructed experimentally and may be particularly useful for a clear observation of an intermediate Higgs boson at hadron colliders. All these new theoretical aspects of the model discussed in this work may lead us

to further theoretical considerations in future and could therefore constitute an additional motivation for us to explore new physics, which may hopefully open up another possibility of investigating the nature of the neutrino particles.

Acknowledgements

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Figure Captions

Fig. 1: *Graphs relevant for the radiatively induced neutrino mass matrix m^ν in the Feynman gauge.*

Fig. 2: *Production cross sections of the Higgs boson as function of its mass at different collision machines: SSC (dashed line), LHC (solid line), 2-TeV e^+e^- collider (dashed-dotted line), 1-TeV e^+e^- collider (dotted line).*

Fig. 3: *The behaviour of the ratio $R = \Gamma(H^0 \to \nu N)/\Gamma(H^0 \to b\bar{b})$ for A=100 GeV and different values of a/A: 0.2 (dotted line), 0.1 (dashed line), 0.05 (solid line).*

Fig. 4: *The branching ratio of Higgs decays into two heavy neutrinos. We have set $a/A = 0.2$ and $A = 150$ GeV (solid line), 200 GeV (dashed line), 250 GeV (dotted line).*

Fig. 5: *The branching ratio of Higgs decays into νN for $A = 200$ GeV and $a/A = 0.1$ (solid line), $A = 200$ GeV and $a/A = 0.2$ (dashed line), $A = 400$ GeV and $a/A = 0.1$ (dotted line), $A = 400$ GeV and $a/A = 0.2$ (dashed-dotted line).*

Fig. 6: *The total cross section of the reaction $pp \to W^{-*} \to l^- N X$ at SSC (solid line) and at LHC (dashed line).*